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LESSON

11.4 Corresponding Parts of Similar Triangles

Is there more to similar triangles than just proportional side lengths and congruent angles? For example, are there relationships between the lengths of corresponding altitudes, corresponding medians, or corresponding angle bisectors in similar triangles? Let's investigate.

Big doesn't necessarily mean better. Sunflowers aren't better than violets.
EDNA FERBER

Investigation 1
Corresponding Parts

You will need

- a compass
- a straightedge

Use unlined paper for this investigation. Have each member of your group pick a different scale factor.

Step 1 Draw any triangle. Using your scale factor, construct a similar triangle of a different size.

Step 2 Construct a pair of corresponding altitudes and use your compass to compare their lengths. How do they compare? How does the comparison relate to the scale factor you used?

Step 3 Construct a pair of corresponding medians. How do their lengths compare?

Step 4 Construct a pair of corresponding angle bisectors. How do their lengths compare?

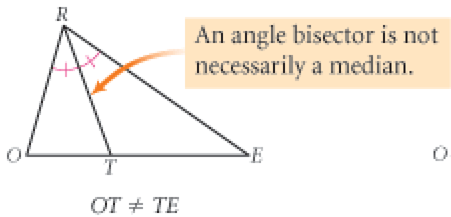
Step 5 Compare your results with the results of others near you. You should be ready to make a conjecture.

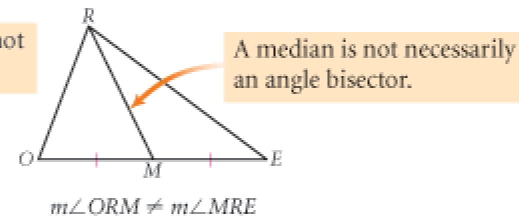
Proportional Parts Conjecture C-94

If two triangles are similar, then the lengths of the corresponding ? , ? , and ? are ? to the lengths of the corresponding sides.

The discovery you made in this investigation may seem intuitive. The example at the end of this lesson will show you how you can prove one part of the conjecture. In the next two exercise sets, you will prove the other parts. Now we'll look at another proportional relationship involving an angle bisector of a triangle.

Recall when you first saw an angle bisector in a triangle. You may have thought that the bisector of an angle in a triangle divides the opposite side into two equal parts as well. A counterexample shows that this is not necessarily true. In $\triangle ROE$, $\overline{RT}$ bisects $\angle R$, but point T does not bisect $\overline{OE}$.





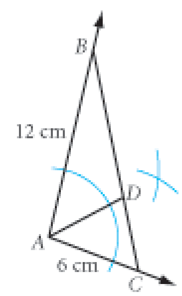
The angle bisector does, however, divide the opposite side in a particular way.

Investigation 2
Opposite Side Ratios

You will need

- a compass
- a ruler

In this investigation you'll discover that there is a proportional relationship involving angle bisectors.



Step 1 Draw any angle. Label it A.

Step 2 On one ray, locate point C so that AC is 6 cm. Use the same compass setting and locate point B on the other ray so that AB is 12 cm. Draw $\overline{BC}$ to form $\triangle ABC$.

Step 3 Construct the bisector of $\angle A$. Locate point D where the bisector intersects side $\overline{BC}$.

Step 4 Measure and compare CD and BD.

Step 5 Calculate and compare the ratios $\frac{CA}{BA}$ and $\frac{CD}{BD}$.

Step 6 Repeat Steps 1–5 with AC = 10 cm and AB = 15 cm.

Step 7 Compare your results with the results of others near you. State a conjecture.

Angle Bisector/Opposite Side Conjecture C-95

A bisector of an angle in a triangle divides the opposite side into two segments whose lengths are in the same ratio as __?__ .

In this lesson you discovered proportional relationships within a triangle and between similar triangles. Do you think that proportional relationships exist between parts of other similar polygons? For example, what do you think might be true about the corresponding diagonals of similar quadrilaterals?

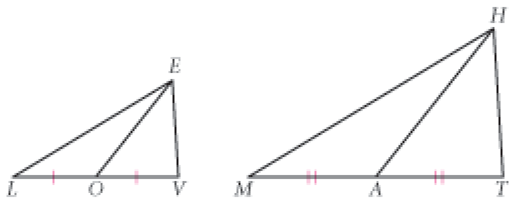
keymath.com/DG

To test your conjectures, see the **Dynamic Geometry Exploration** Similar Polygons at www.keymath.com/DG .

Now let's look at how you can use deductive reasoning to prove one part of the Proportional Parts Conjecture that you discovered through inductive reasoning earlier in this lesson.

EXAMPLE Prove that the lengths of the corresponding medians of similar triangles are proportional to the lengths of the corresponding sides.

▶ Solution Consider similar triangles $\triangle LVE$ and $\triangle MTH$ with corresponding medians $\overline{EO}$ and $\overline{HA}$. You need to show that the lengths of corresponding medians are proportional to the lengths of corresponding sides. For instance, you can show that $\frac{EO}{HA} = \frac{EL}{HM}$. Using the think backward reasoning strategy, you can see that similar triangles will help you prove that side lengths are proportional. If you accept the SAS Similarity Conjecture as true, you can show $\frac{EO}{HA} = \frac{EL}{HM}$ by first proving that $\triangle ELO \sim \triangle HMA$.



1. $\overline{EO}$ and $\overline{HA}$ are medians — Given
2. O is midpoint of $\overline{LV}$; A is midpoint of $\overline{MT}$ — Definition of median
3. $LO = OV$; $MA = AT$ — Definition of midpoint
4. $\triangle LVE \sim \triangle MTH$ — Given
5. $\frac{EL}{HM} = \frac{LV}{MT}$ — Definition of similar polygons
6. $\frac{EL}{HM} = \frac{LO + OV}{MA + AT}$ — Segment addition
7. $\frac{EL}{HM} = \frac{LO + LO}{MA + MA}$; $\frac{EL}{HM} = \frac{2LO}{2MA}$; $\frac{EL}{HM} = \frac{LO}{MA}$ — Substitution and algebra
8. $\angle L \cong \angle M$ — Definition of similar polygons
9. $\triangle ELO \sim \triangle HMA$ — SAS Similarity Conjecture
10. $\frac{EO}{HA} = \frac{EL}{HM}$ — Definition of similar polygons

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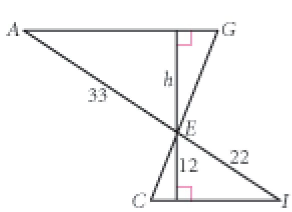
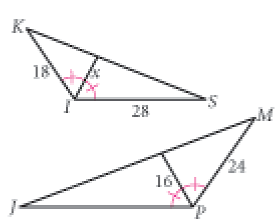
EXERCISES

For Exercises 1–13, use your new conjectures. All measurements are in centimeters.

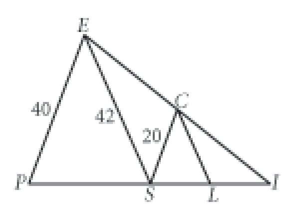
1. $\triangle ICE \sim \triangle AGE$
h = ?



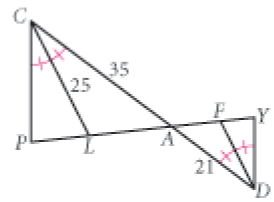
2. $\triangle SKI \sim \triangle JMP$
x = ?



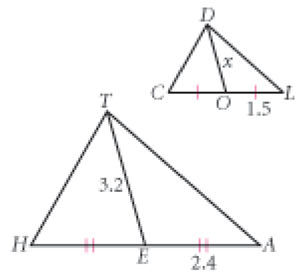
3. $\triangle PIE \sim \triangle SIC$
Point S is the midpoint of $\overline{PI}$. Point L is the midpoint of $\overline{SI}$. CL = ? ⓗ



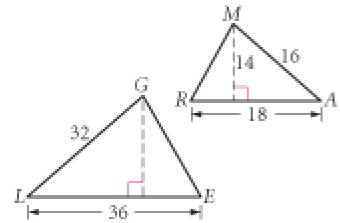
4. $\triangle CAP \sim \triangle DAY$
$FD = \underline{?}$



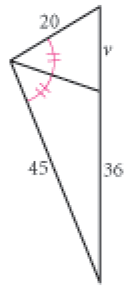
5. $\triangle HAT \sim \triangle CLD$
$x = \underline{?}$



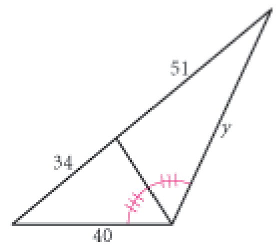
6. $\triangle ARM \sim \triangle LEG$
Area of $\triangle ARM = \underline{?}$
Area of $\triangle LEG = \underline{?}$



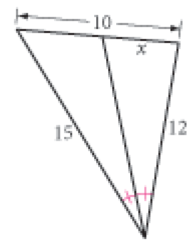
7. $v = \underline{?}$



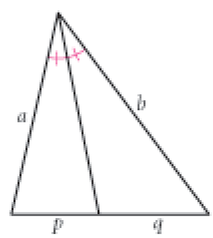
8. $y = \underline{?}$



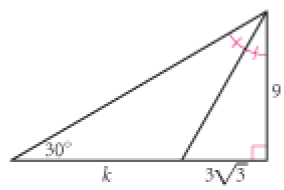
9. $x = \underline{?}$ ⓗ



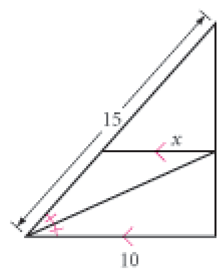
10. $\frac{a}{b} = \underline{?}$, $\frac{a}{p} = \underline{?}$



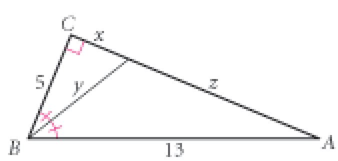
11. $k = \underline{?}$



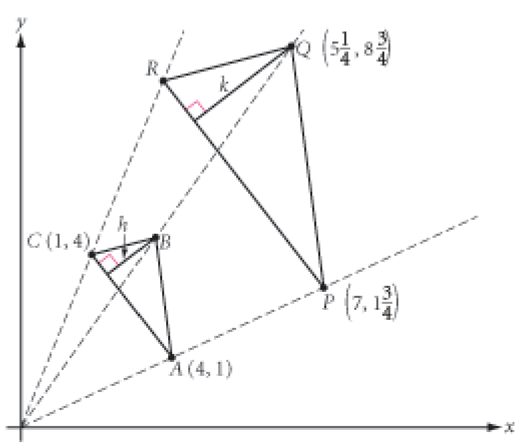
12. $x = \underline{?}$ ⓗ



13. $x = \underline{?}$, $y = \underline{?}$, $z = \underline{?}$ ⓗ



14. Triangle *PQR* is a dilated image of $\triangle ABC$. Find the coordinates of *B* and *R*. Find the ratio $\frac{k}{h}$.



15. Aunt Florence has willed to her two nephews a plot of land in the shape of an isosceles right triangle. The land is to be divided into two unequal parts by bisecting one of the two congruent angles. What is the ratio of the greater area to the lesser area?

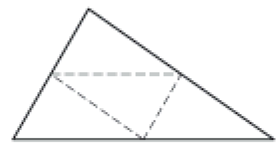
16. ***Developing Proof*** Prove that corresponding angle bisectors of similar triangles are proportional to corresponding sides. ⓗ

Review

17. Use algebra to show that if $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$. ⓗ

18. A rectangle is divided into four rectangles, each similar to the original rectangle. What is the ratio of short side to long side in the rectangles?

19. ***Developing Proof*** In Chapter 5, you discovered that when you construct the three midsegments in a triangle, they divide the triangle into four congruent triangles. Are the four triangles similar to the original? Explain why.



20. A rectangle has sides a and b. For what values of a and b is another rectangle with sides $2a$ and $\frac{b}{2}$

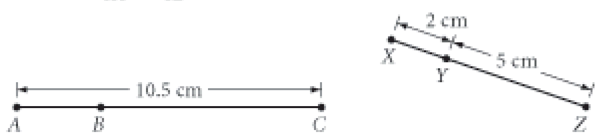
a. Congruent to the original?

b. Equal in perimeter to the original?

c. Equal in area to the original?

d. Similar but not congruent to the original?

21. Assume $\frac{AB}{XY} = \frac{BC}{YZ}$. Find AB and BC.



IMPROVING YOUR ALGEBRA SKILLS

Algebraic Magic Squares II

In this algebraic magic square, the sum of the entries in every row, column, and diagonal is the same. Find the value of x.

$8 - x$	15	14	$11 - x$
12	$x - 1$	x	9
8	$x + 3$	$x + 4$	5
$2x - 1$	3	2	$2x + 2$